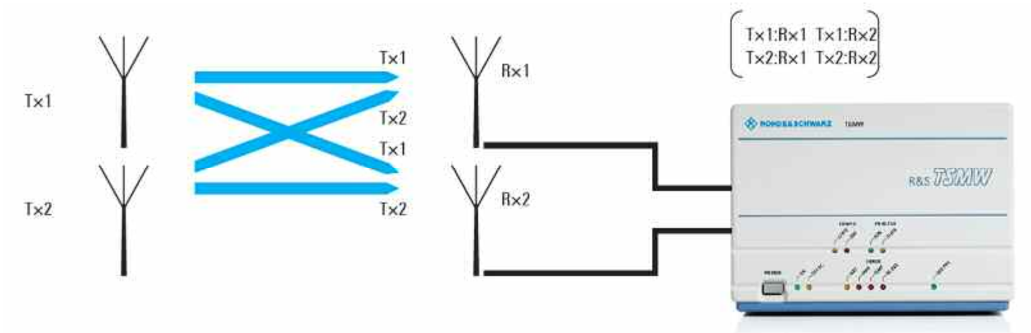
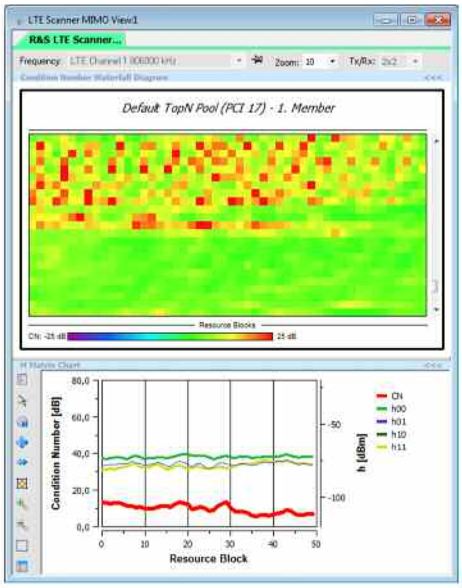
Assessing a MIMO Channel

White Paper





MIMO technologies are an essential component of state-of-the-art mobile radio systems and are key to achieving extremely ambitious capacity goals that include providing stable data rates in the two- to three-digit Megabit per second range over a broad coverage area. However, the effectiveness of these technologies is not always guaranteed. A channel state matrix can provide the information needed to determine whether spatial multiplexing is possible for multilayer data transmission. Simple indicators for evaluating the mobile radio channel are derived from the channel matrix. This white paper presents these parameters and discusses them based on simulated and real coverage measurement results.

1SP18

Stefan Schindler, Heinz Mellein

February 2011-0E

ROHDE&SCHWARZ

Contents

1 Introduction

Multiple input multiple output (MIMO) technologies are an essential component of state-of-the-art mobile radio systems – such as HSPA+[1] and LTE[2] – and are key to achieving extremely ambitious capacity goals that include providing stable data rates in the two- to three-digit Megabit per second range over a broad coverage area. The use of spatial multiplexing allows the data rates to be multiplied without requiring additional bandwidth or increasing the overall transmit power. However, the effectiveness of these technologies is not always guaranteed. The MIMO channel state information (CSI) can be used to determine whether spatial multiplexing is practicable. For example, under certain circumstances (such as identically polarized radio waves), spatial multiplexing does not bring any benefit with respect to capacity even with a direct line of sight (LOS) between the transmitter and the receiver, while it is of much more benefit in extreme cases of multipath propagation (such as indoor reception).

Assessing the status of a MIMO channel requires a continuous evaluation of complex matrices. Fortunately, relatively simple mathematical indicators, such as the channel rank and condition, can be interpreted for a rapid assessment of the MIMO channel. This paper describes these parameters and interprets them as they relate to the MIMO channel state matrix. The theory is further verified based on examples simulated in the lab as well as actual measurement results from a live LTE mobile radio network.

[1] High Speed Packet Access
[2] Long Term Evolution

2 What is MIMO about?

From a purely technical standpoint, the most important consideration in the specification of mobile data transmission systems is the optimum utilization of the following two transmission characteristics for the mobile radio channel:

1. **Channel Capacity**,
   i.e. the maximum transmittable data rate, measured as bits per second (Bit/s)
2. **Performance**,
   i.e., minimizing the probability of transmission errors (displayed as bit or block error rate, for example)

The (theoretically) maximum channel capacity $\boldsymbol{C}$ for a channel that is subject to additive noise can be calculated using the bandwidth $\boldsymbol{B}$ and the effective signal-to-noise ratio (SNR), i.e. the quotient of the received signal power ***S*** and the noise power ***N***, in accordance with the Shannon-Hartley theorem (refer to [1][4][5][6][10]). The transmission capacity of binary data in bit/s can thus be expressed as follows:

$$\boldsymbol{C = B \cdot log_2(1 + \frac{S}{N})}$$

The available channel bandwidth $\boldsymbol{B}$ has a directly proportionate effect on the channel capacity and is therefore the deciding factor for achieving the targeted peak data rates. That is why advanced mobile radio systems such as LTE primarily use an increase in bandwidth (10 or 20 MHz, or even up to 100 MHz in the future with LTE-A[3]) to achieve the promised data rates (the goal for LTE-A is a net data rate of 1 Gigabit/s [9]). In comparison: a GSM mobile radio channel occupies about 200 kHz only.

Particularly for mobile radio, the signal-to-noise ratio is a function of the physical distance between the receiver (such as a smart phone) and the base station, and thus is an extremely volatile and location-dependent value. The SNR is decisive for the transmission quality (i.e., performance) and therefore for the actual data throughput in a radio cell.

A thoughtfully designed MIMO system can positively affect these two channel attributes without increasing the bandwidth or transmit signal power: The performance can be improved with transmit and/or receive antenna diversity, while spatial multiplexing will serve to increase the channel capacity. This means that spatial multiplexing alone will make the transmission of multilayer data signals possible, leading to a significant increase in the channel capacity. The general calculation of the MIMO channel capacity then becomes very complicated as compared to the above Shannon-Hartley theorem for single input single output (SISO). As a simple rule of thumb, however, the greatest possible MIMO channel capacity for an M-layer data transmission can be estimated as M-times the SISO channel capacity as calculated by Shannon. More detailed calculations are available in [10], among other resources.

3LTE Advanced

2.1 Using diversity to improve performance

Performance can be improved by adding transmit and/or receive antennas. In the case of receive diversity, multiple receive antennas "collect" additional signal power ***S*** . The effective signal-to-noise ratio (SNR) can then be improved by using intelligent signal processing, for example maximum ratio combining (MRC), a process that maximizes the SNR.

The same effect can be achieved by adding transmit antennas (transmit diversity), with all transmit antennas broadcasting the same user data stream. With dedicated pre-coding, the radio channel characteristics are already taken into consideration on the transmitter side, with the result that these characteristics are essentially compensated prior to the transmission of the signal. However, this requires that the transmitter (such as a base station) knows the channel state, which in turn requires that sufficient feedback be provided by the receiver (such as a mobile station), particularly in FDD systems where there are different transmit and receive frequency bands. This is considered a ***closed loop*** transmit diversity scheme. To minimize the feedback effort, mobile radio systems such as LTE use a variety of predefined pre-coders. Based on the current receive situation, the receiver requests one of the known pre-coders from the transmitter in the form of a pre-coding matrix indicator (PMI).

If sufficient transmit antennas are available, beam-forming can be used to line up the transmit signal directly with the receiver, providing a particularly effective method of improving the available SNR. Beam-forming makes sense only if the transmitter knows the precise location of the receiver; otherwise, the process will fall short of its objective, both literally and figuratively. Effective tracking of the beam to a mobile radio receiver that is typically in motion requires that a very detailed channel status report for the mobile station be provided to the base station in real time, causing considerable signaling traffic on the reverse link. For this reason, this method is most effective in stationary operation; in other words, for a receiver that moves little or not at all. IEEE 802.11n specifies this method as a "calibration procedure" for WLAN broadband radio.

Although ***open loop*** transmit diversity schemes do not require a response from the receiver, they are by nature not nearly as effective. So for example, it is under only the absolute best conditions that adding a second transmit antenna will provide an increase in performance of up to 3 dB, or double the effective SNR. The most popular of these methods is based on a space time block coding (STBC) or space frequency block coding (SFBC). These MIMO technologies improve performance in particular with respect to the radio cell boundaries, in other words they effectively increase the size of the radio cell. Although these methods do not increase the channel capacity with respect to the peak data rate, they do improve the effective throughput per radio cell as a result of the improved performance.

Refer especially to [2] for more on these concepts and for related algorithms with a focus on commercial mobile radio systems.

2.2 Increasing the peak data rate by spatial multiplexing

MIMO is used in conjunction with spatial multiplexing as a way to increase the peak data rate. This method uses additional transmit **and** receive antennas to transmit parallel data streams. As a result, the channel capacity is increased without increasing the bandwidth or the SNR. With an M_S x M_R MIMO constellation – where M_S is the number of transmit antennas and M_R is the number of receive antennas – the maximum number of spatially separate data streams is defined as $M = \min\{M_S, M_R\}$. This means that just like a 2 x 2 MIMO system, a 4 x 2 MIMO system can spatially multiplex a maximum of 2 data layers (i.e. independent data streams). A 4 x 2 MIMO system can in fact do both: It can double the data rate through spatial multiplexing while simultaneously increasing the performance by means of transmit diversity as described in the previous section.

It is this spatial multiplexing that makes multiple data layer transmission possible. These methods are truly "MIMO ready" because they allow multiple data streams to be transported simultaneously via the same frequency band, in contrast to the antenna diversity schemes from the previous section that "merely" improve performance.

The question of whether a spatial division of independent data streams actually works depends on the antenna geometry and thus on the correlation (a measure of the mutual influence) of the spatially separate signals. The correlation can be eliminated by means of orthogonal (cross) polarization of the antennas, or by ensuring that they are sufficiently far apart, although the latter is not easily accomplished in compact terminal equipment.

In the end, though, it is the radio channel state that plays the decisive role with respect to MIMO suitability, and so the following section provides a discussion of the channel state based on the simplest example of a 2 x 2 MIMO channel.

3 Assessing a MIMO channel

An appropriate mathematical model is needed to assess the MIMO characteristics of a mobile radio channel. The simplified model consists essentially of a set of linear equations. The transmit and receive antennas are represented by a transmit and a receive signal vector, respectively. The actual transmission characteristic, or the current channel state, is summarized in a matrix. It is this channel state matrix that is useful in assessing the MIMO channel, and therefore is described further here.

3.1 Description of the MIMO channel in a matrix

OFDM divides the used signal bandwidth $\boldsymbol{B}$ into narrow-band sub channels whose bandwidth $\boldsymbol{\Delta f}$ (e.g. 15 kHz for LTE) does not exceed the coherency bandwidth $\boldsymbol{B_c}$ of the mobile radio channel under consideration. Under this restriction, the transmitted signal is merely distorted by a non-frequency-selective attenuation and phase shift (commonly known as *flat fading*), which corresponds mathematically to multiplication by a complex number. If this complex transmission coefficient is calculated for every OFDM sub channel in the receiver – for example, based on known reference or pilot signals – one-tap equalization can be achieved by multiplying every sub channel with the inverse of its transmission coefficient. However, the signal is still affected by the unavoidable, additive overlay of thermal noise and inband radio interference, making it difficult to determine the transmission coefficients exactly and in fact providing only a certain degree of accuracy. This is one of the main reasons why the process of determining these transmission coefficients is called ***channel estimation***.

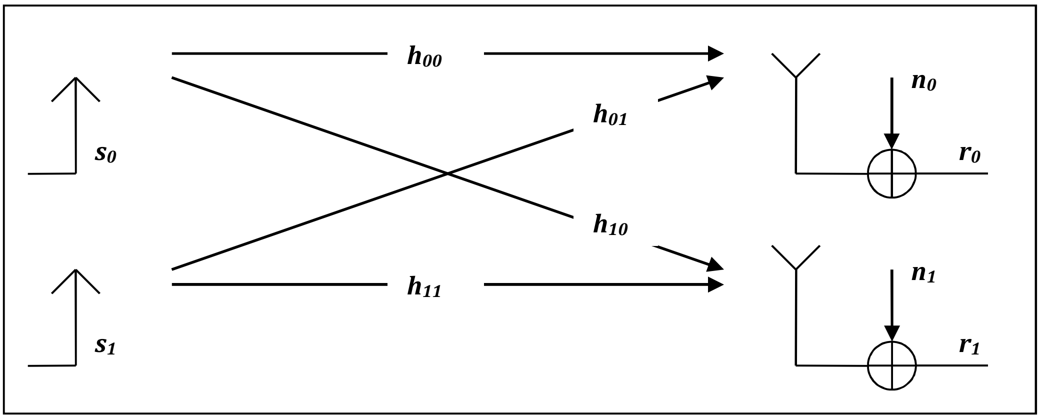


Figure 1: 2 x 2 MIMO transmission model

During spatial multiplexing for the purpose of increasing the transmission capacity, multiple data layers are transmitted simultaneously in the same frequency range via multiple transmit antennas to multiple receive antennas. In the simplest scenario, two data streams are transmitted via two transmit antennas to two receive antennas (see *Figure 1*). In contrast to SISO, this generates 4 individual transmission paths, each with one complex transmission coefficient (per OFDM sub channel). The resulting 2 x 2 MIMO transmission channel can be represented mathematically as a 2 x 2 matrix with 4 complex-valued matrix elements. Each of the two receivers estimates two of the channel matrix elements based on known pilot or reference signals.

This 2 x 2 MIMO transmission as shown in *Figure 1* can be described with two linear equations:

$$\boldsymbol{r_0 = h_{00} \cdot s_0 + h_{01} \cdot s_1 + n_0}$$
$$\boldsymbol{r_1 = h_{10} \cdot s_0 + h_{11} \cdot s_1 + n_1}$$

where s_j is the transmit signal from the *j*-th transmit antenna and r_i is the receive signal at the *i*-th receive antenna. The factors h_{ij} identify the complex transmission coefficient from the *j*-th transmit antenna to the *i*-th receive antenna. n_i reflects the additive noise in the *i*-th receiver. In matrix format, this is represented as follows:

$$\begin{pmatrix} \boldsymbol{r_0} \\ \boldsymbol{r_1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{h_{00}} & \boldsymbol{h_{01}} \\ \boldsymbol{h_{10}} & \boldsymbol{h_{11}} \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{s_0} \\ \boldsymbol{s_1} \end{pmatrix} + \begin{pmatrix} \boldsymbol{n_0} \\ \boldsymbol{n_1} \end{pmatrix}$$

or, in short form:

$$\boldsymbol{r = H \cdot s + n}$$

It is the receiver's job is to solve this equation. The receive vector $\boldsymbol{r}$ is known and the transmit vector $\boldsymbol{s}$ must be determined. To solve the equation, the channel matrix $\boldsymbol{H}$ must be estimated. To demonstrate, the transmission coefficients h_{ij} can be interpreted with $i \neq j$ as crosstalk.

The solvability of this equation can be assessed using the channel matrix. If the equation is “satisfactorily” solvable, then this channel can be used for multilayer signal transmission. If the equation is unsolvable or only unsatisfactorily solvable, then the use of spatial multiplexing makes no sense for this channel. The answer to this question determines the condition and rank of this matrix.

3.2 The condition and rank of a matrix

The mathematical concept of ***condition*** characterizes how sensitive the solution to a problem is to imprecise or faulty input data. For example, the ***condition of a matrix*** is an indicator of how well the linear equation that is described with this matrix can be solved. Translated to the problem discussed here of multilayer signal transmission via a MIMO channel, the receiver detects the actual transmit signal via the estimated channel matrix. However, the receive signal – that is, the input values for the equation to be solved – is disrupted by at least additive noise. The possible receive quality, i.e. how reliably the multilayer transmit signal could be reconstructed, is thus dependent on the condition of the channel matrix. A well-conditioned channel matrix allows reliable multilayer reception. An ill-conditioned matrix prevents this or at the very least makes it difficult.

As a result, a calculation of the condition for the estimated channel matrix provides an important indication of whether of the MIMO channel under consideration can be spatially multiplexed. The general definition of the condition of a matrix is based on its singular values described in the next section.

The condition or ***condition number κ(H)*** (*CN*) of matrix $\boldsymbol{H}$ is calculated as follows:

$$\boldsymbol{\kappa(H) = \frac{\sigma_{max}}{\sigma_{min}} \geq 1}$$

where σ_{max} is the largest singular value and σ_{min} is the smallest singular value in matrix H.

The rank of this matrix is the number of singular values not equal to zero. The rank of the channel matrix is thus an indicator of how many data streams can be spatially multiplexed on the MIMO channel.

The rank and condition of a channel matrix also exist for every possible matrix dimension and thus for every possible MIMO constellation! This means that these parameters can be used to characterize not only 2 x 2 MIMO channel states, but also 4 x 2, 4 x 4 or even 8 x 8 MIMO constellations! Thus, all kinds of MIMO configurations we can expect in mobile communications next, can be assessed by these figures.

The following section explains what the rank and condition of the channel matrix say about the suitability of the channel for spatial multiplexing based on the singular value decomposition of a matrix.

3.3 Singular value decomposition of a channel matrix

For the purpose of singular value decomposition (SVD) of the channel matrix, H is formulated as follows [1]:

$$\boldsymbol{H} = \boldsymbol{U} \cdot \boldsymbol{\Sigma} \cdot \boldsymbol{V}^{\boldsymbol{H}}$$

The columns in matrix U and V are defined by the eigenvectors of $H \cdot H^H$ and $H^H \cdot H$, respectively, i.e. they can be calculated purely from the channel matrix (H^H represents the Hermitian matrix of H, i.e. the conjugate transpose of complex matrix H). Matrix Σ contains only the singular values σ_i from channel matrix H on the principle diagonal and is otherwise *0*; that is, it applies for a 2 x 2 channel matrix H:

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_1 \end{pmatrix}$$

As shown in *Figure 2*, the singular value decomposition of the channel matrix can be seen as the separation of the MIMO channel into two crosstalk-free transmission channels with transmission coefficients σ_0 and σ_1; in other words, it is split into two virtual, parallel SISO channels. In accordance with the Shannon-Hartley theorem, each of these virtual SISO channels will contribute to the total capacity of the MIMO channel as long as transmission coefficient σ_i is a sufficiently large value, i.e., as long as sufficient “spatial” power is transmitted.

This type of singular value decomposition of the channel matrix makes a more appropriate model possible for signal transmission via the MIMO channel: Transmit signal vector s is first transformed using matrix U into the orthogonal space expanded by Σ. The transformed signal vector components are transmitted via singular value matrix Σ. Matrix V is then used for the reverse transformation to the original signal vector space. As a result, the crosstalk does not “disappear” from the signal path in this transformed model, but rather is simply hidden away in transformation matrices U and V.

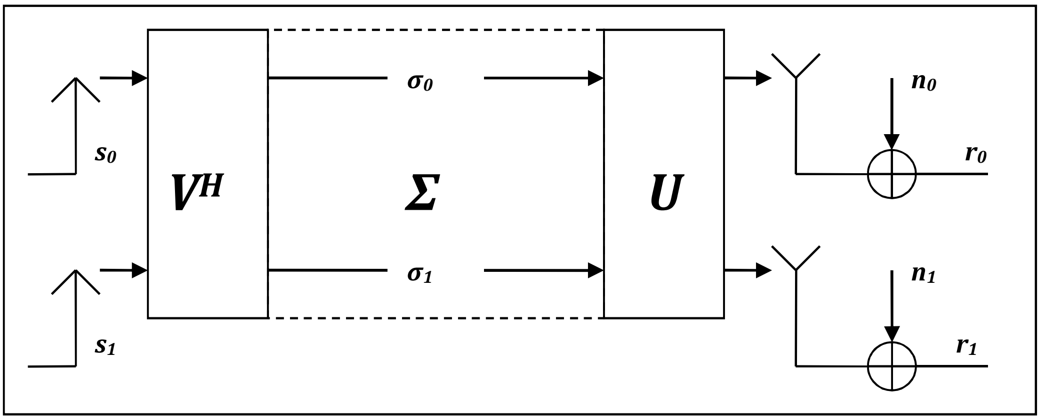


Figure 2: Singular value decomposition of a 2 x 2 MIMO transmission channel

On the one hand, this interpretation of the singular value decomposition makes it clear that the number of data streams being multiplexed cannot exceed the number of adequately sized singular values in the channel matrix – in other words, the rank of the channel matrix is the deciding indicator in this respect. On the other hand, the optimal scenario for spatial multiplexing would probably be if the magnitudes of all singular values (the two singular values $\boldsymbol{\sigma_0}$ and $\boldsymbol{\sigma_1}$ in our 2 x 2 example) were approximately equal. If one of these values is much greater than the other, it becomes very difficult to decode the "weaker" path so that it is still usable. This underscores the importance of the ratio of the singularity values to one another, which brings us back to the condition number for the channel matrix again. The closer the ratio between the largest and the smallest singular value – so in other words the condition number for the matrix – is to 1, the better suited the MIMO channel is for spatial multiplexing. And the larger this condition number is for the channel matrix, the less sense that multilayer transmission makes. For the sake of completeness, it must be said that this singular value decomposition, and thus the determination of the rank and condition of the MIMO channel matrix, can also be performed for higher order multiple antenna constellations – although the effort required for the calculations rises significantly.

3.4 Correlation matrix and eigenvalues

If one assumes – perfectly valid in flat fading conditions – a stable radio channel over time, at least for the duration of an estimation period, along with purely additive and normally distributed distortions, then the singular values $\boldsymbol{\sigma_i}$ from a matrix $\boldsymbol{H}$ will equal the square root of the eigenvalues $\boldsymbol{\lambda_i}$ of the associated correlation matrix $\boldsymbol{H^H H}$; in other words, $\boldsymbol{\sigma_i^2 = \lambda_i}$ applies [10]. This means that it is also possible to determine the rank and condition number of the channel matrix using eigenvalue decomposition.

Practically, the condition number CN is given in logarithmic form as follows:

$$\boldsymbol{CN/_{dB} = 20 \cdot log_{10}\kappa(H) = 20 \cdot log_{10}\frac{\sigma_{max}}{\sigma_{min}} = 10 \cdot log_{10}\frac{\lambda_{max}}{\lambda_{min}} \geq 0dB}$$

3.5 Channel condition number and channel capacity

The Shannon-Hartley theorem states that the bandwidth and SNR are the deciding factors with respect to channel capacity. However, as illustrated in *Figure 3*, the condition of the channel state matrix is also a contributing factor in MIMO operation. For example, assuming a spectral efficiency of 10 bit/s/Hz, a condition number of 0 dB would require a SNR of 15 dB. This increases to approx. 20 dB if the condition number worsens to approx. 16 dB. In MIMO operation, therefore, it is not sufficient to consider SNR only when assessing the channel capacity. It is just as important to look at the condition of the MIMO channel status matrix.

Figure 3 clearly shows the capacity improvements possible with MIMO. A well-conditioned MIMO channel state matrix will provide a high spectral efficiency even at low SNRs. The theoretical foundation for *Figure 3* is found in [10].

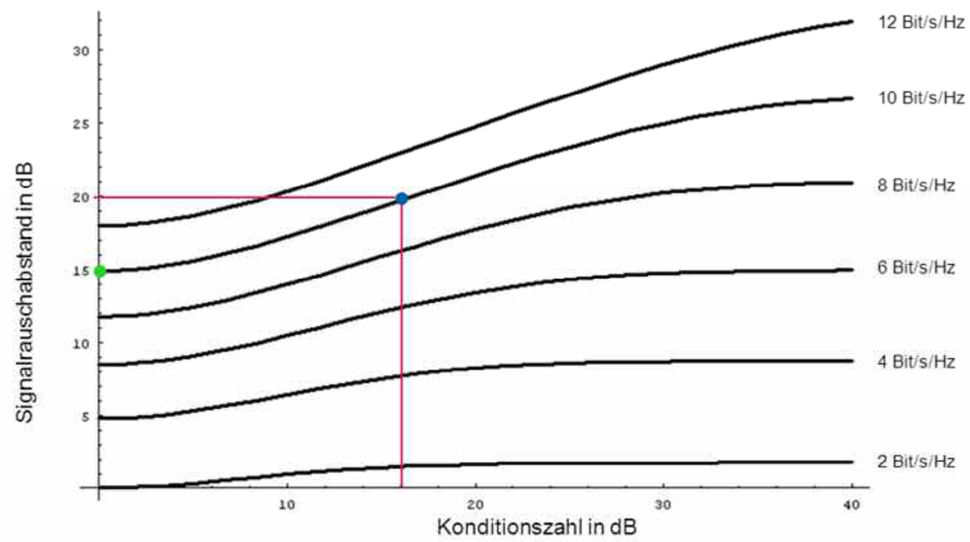


Figure 3: Signal-to-noise ratio and condition number

Figure 3 also illustrates that the “positive” effect of the condition number will disappear at a spectral efficiency of e.g. 4 bit/s/Hz and an SNR of about 20 dB, or at a spectral efficiency of only 2 bit/s/Hz when the SNR is around 15 dB. This is why in real-world operation, logarithmic condition numbers of 0 – 10 dB are considered to be very good, while any channel with a condition number above 20 dB is considered to be unusable for MIMO.

4 Simulated examples

Take for example the purely theoretical case of a channel matrix with zero crosstalk; in other words, two parallel SISO channels:

$$\boldsymbol{H} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

The equation is very simple:

$$\boldsymbol{r_0} = \boldsymbol{s_0} + \boldsymbol{n_0}$$
$$\boldsymbol{r_1} = \boldsymbol{s_1} + \boldsymbol{n_1}$$

The two singular values $\boldsymbol{\sigma}_{\mathbf{max}}$ and $\boldsymbol{\sigma}_{\mathbf{min}}$ in this matrix are 1. This returns a rank of 2 and a condition number of 1 (logarithmic 0 dB), making it, as expected, an ideal candidate for spatial multiplexing. However, this is a theoretical best-case scenario that would never occur on a real mobile radio channel.

A second extreme theoretical scenario is completely plausible: A phase-equivalent, symmetrical division of the signal power from one transmit antenna to both receive antennas; in other words, the worst-case crosstalk scenario. Something close to this scenario is seen when a direct, unobstructed LOS exists between a base station and the mobile station. For this scenario, $\boldsymbol{H}$ is as follows:

$$\boldsymbol{H} = \begin{pmatrix} \mathbf{1}/\sqrt{\mathbf{2}} & \mathbf{1}/\sqrt{\mathbf{2}} \\ \mathbf{1}/\sqrt{\mathbf{2}} & \mathbf{1}/\sqrt{\mathbf{2}} \end{pmatrix}$$

The square values of two matrix elements always add up to 1, i.e. the power from one transmit antenna is divided, with one half going to a direct component and the other half to a crosstalk component. With singular value decomposition, this results in a $\boldsymbol{\sigma}_{\mathbf{max}}$ value of $\sqrt{\mathbf{2}}$ and a $\boldsymbol{\sigma}_{\mathbf{min}}$ value of 0. The rank of this channel matrix is 1, and a condition number is either undefined or approaches infinity as a result of the division by zero. In other words, an unobstructed LOS between the transmitter and the receiver is not suited to spatial multiplexing! This is because the receive antenna is not capable of separating the two transmit signals. Does this make MIMO impractical along an open highway, for example?

To answer this question, consider a third example based on the same scenario: The signal power to the receive antennas is divided symmetrically. In this case, however, the crosstalk signals are phase-shifted by 90°, as would occur with a longer transmission path, for example. The channel matrix would be as follows:

$$\boldsymbol{H} = \begin{pmatrix} \mathbf{1}/\sqrt{\mathbf{2}} & \boldsymbol{j}\cdot\mathbf{1}/\sqrt{\mathbf{2}} \\ \boldsymbol{j}\cdot\mathbf{1}/\sqrt{\mathbf{2}} & \mathbf{1}/\sqrt{\mathbf{2}} \end{pmatrix}$$

This orthogonality of the direct component and the crosstalk component makes it easy for the receiver to separate the two transmit signals. This is once again reflected in the singular values $\boldsymbol{\sigma}_{\mathbf{max}}$ and $\boldsymbol{\sigma}_{\mathbf{min}}$ for this channel matrix. As in the first example, these are both 1, but in this case the condition number is also 1, making the channel ideal for spatial multiplexing.

It is relatively easy to create orthogonal crosstalk paths by using cross-polarized antennas, allowing spatial multiplexing to be used successfully even with an unobstructed LOS between the base and mobile station. This cross-polarization is widely used in commercial mobile radio networks, and is almost mandatory for successful MIMO operation, as illustrated here!

Figure 4 shows condition number results from the lab using coverage measurement equipment from Rohde & Schwarz. The measurements show a 10 MHz LTE signal for which the condition number per resource block (equal to 180 kHz) is determined and displayed graphically (red line). The graph at the upper left shows the discussed worst case with the maximum, phase-synchronous crosstalk and a correspondingly unfavorable condition number of greater than 40 dB. The graph at the upper right shows an essentially perfect condition number of 0 dB over the entire signal bandwidth with an orthogonal crosstalk (for example, created using cross-polarized antennas and a direct LOS). In both examples, the simulated SNR was greater than 20 dB. In the third measurement graph at the bottom left, the SNR was reduced to 10 dB. However, the measured condition number remains in the acceptable range of under 10 dB.

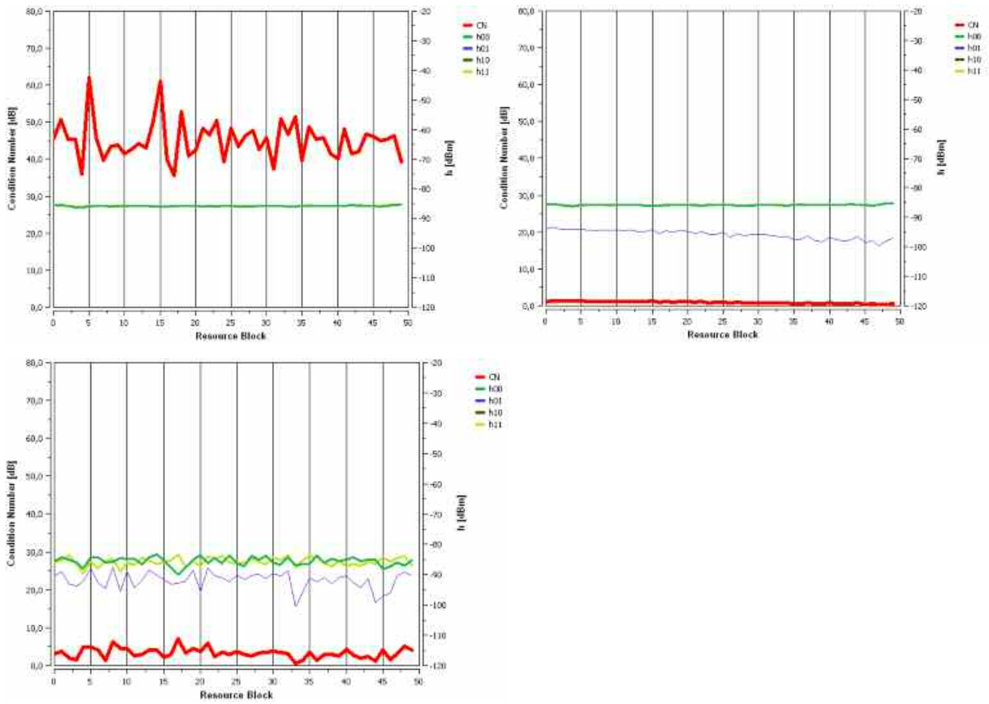


Figure 4: Measurement results for the condition number

A final, more realistic, example includes a dominant direct component h_{ii} , but with measurable crosstalk h_{ij}. $\boldsymbol{H}$ is defined as

$$\boldsymbol{H} = \begin{pmatrix} \mathbf{0.9 + 0.1}\boldsymbol{j} & \mathbf{0.2 - 0.3}\boldsymbol{j} \\ \mathbf{-0.3 + 0.5}\boldsymbol{j} & \mathbf{-0.7 - 0.1}\boldsymbol{j} \end{pmatrix}$$

In this example, the instantaneous value for $\boldsymbol{\sigma_{max}}$ is 1.1457 and for $\boldsymbol{\sigma_{min}}$ is it 0.6906. This is equal to rank 2, and given a condition number $\boldsymbol{\kappa(H)}$ = 1.6583 (logarithmic 4.4 dB), this channel is in fact suited for spatial multiplexing.

5 Field measurements in the real LTE network

The R&S®ROMES coverage analysis software and an R&S®TSMW dual-channel wideband scanner can be used to measure and geographically pinpoint the downlink 2 x 2 MIMO channel during active LTE mobile radio network operation (read more on this topic at www.drivetest.rohde-schwarz.com).

Figure 5 shows the influence of the antenna geometry. The figure to the left shows a measurement using two parallel dipole antennas, while one cross-polarized receive antenna was used in the figure to the right. The difference is significant: Without polarization, the condition number is unusable for MIMO, in contrast to the measurement using a cross-polarized receive antenna. This is confirmed by real-world measurements: Cross-polarized antenna geometries in both base and mobile stations are an important factor for successful MIMO operation.

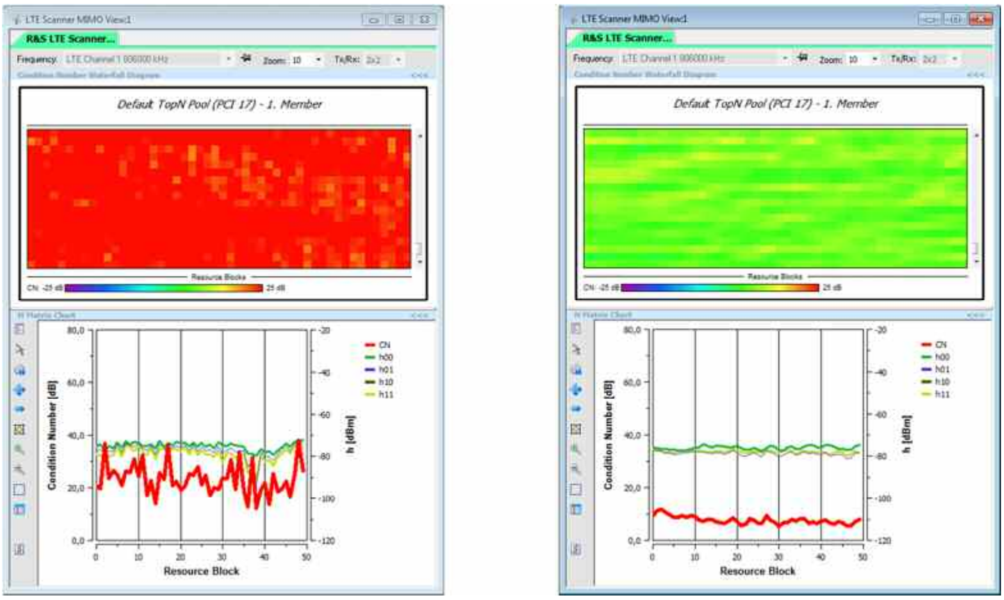


Figure 5: Field measurement with varying antenna geometries

Figure 6 compares two measurements with line-of-sight (LOS) to an LTE base station and with an indirect or obstructed (non-) line-of-sight (NLOS) to the same station. In the LOS operation (left), cross-polarized transmit and receive antennas ensure a good and, more importantly, frequency-dependent MIMO channel condition. Although the MIMO channel condition is no worse in the NLOS operation (driving through a forest, right), it is frequency-selective as a result of the multipath propagation.

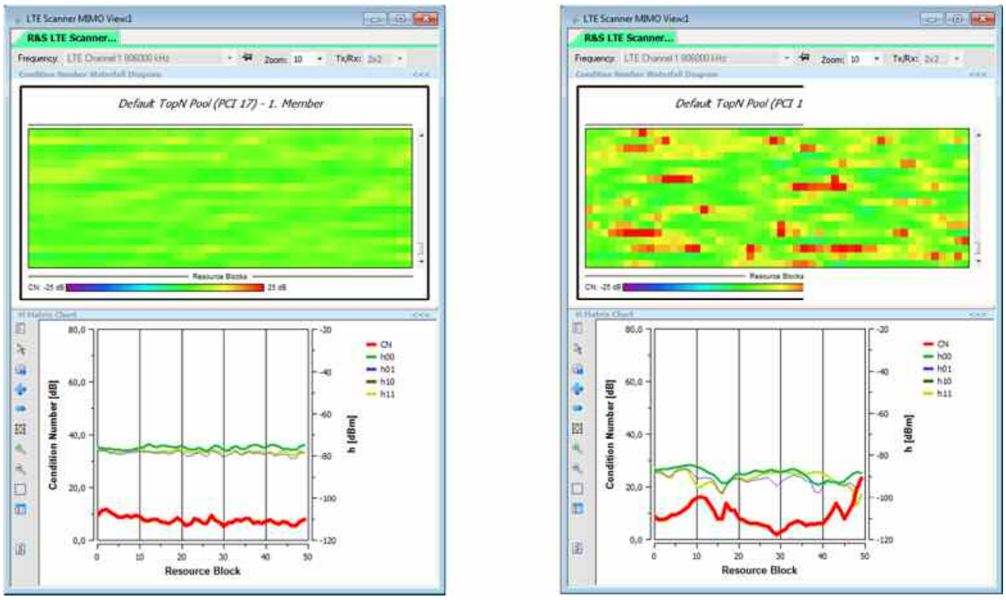


Figure 6: Field measurement with a direct and an indirect LOS link

6 Summary

The rank and condition number of the channel matrix are important for the assessment of the spatial multiplexing capability of a MIMO channel. Both indicators are derived from the singular values of the channel state matrix, which are in turn obtained by means of singular value decomposition (SVD) of the channel matrix. The rank of the matrix is the number of all singular values that have not dropped from the matrix, and the matrix condition number is the ratio of the maximum to the minimum singular value. If the rank of the channel matrix is at least 2, then the MIMO channel is essentially capable of spatial multiplexing. The quality of the spatial multiplexing capability is then additionally quantified by the channel matrix condition number. In practice, a channel matrix having a logarithmic condition number of $20\log(\kappa(H)) < 10$ dB is clearly suitable for spatial multiplexing.

This white paper explains and discusses the parameters for assessing a matrix and thus a MIMO channel based on the simplest example of a 2 x 2 spatial multiplex transmission on the requisite 2 x 2 MIMO channel. The rank and condition number of a matrix are defined over all possible matrix dimensions and thus can also be used for more complex MIMO systems. As a result, the next generation of mobile radio standards, such as LTE-A, specify up to 8 x 8 spatial multiplexing [9], which of course is a transmission of 8 parallel data streams via 8 transmit and 8 receive antennas. But even with an 8 x 8 channel state matrix having 64 complex matrix elements, the rank and condition number still provide the decisive indicators with respect to MIMO characteristics. The same applies to non-quadratic channel matrices, such as a 4 x 2 transmission in which 2 x 2 spatial multiplexing is combined with a performance-enhancing transmit diversity. These types of constellations are likewise specified in the latest mobile radio and broadband radio standards and are already being implemented. Thus, the assessment of the MIMO channel by its channel matrix rank and condition is both, simple and future proofed.

7 References

[1] Kammeyer, "Nachrichtenübertragung" (Communications), 3rd edition, Teubner Verlag

[2] Rohde & Schwarz: "Introduction to MIMO", Application Note 1MA142, 2009

[3] Rohde & Schwarz: "LTE beamforming measurements", Application Note 1MA187, 2011

[4] Dahlman, Parkvall, Sköld, Beming, "3 G Evolution HSPA and LTE for Mobile Broadband", Second edition, Academic Press, 2008

[5] Fazel, Kaiser: "Multi-Carrier and Spread Spectrum Systems", Wiley, 2003

[6] Bahai, Saltzberg, Ergen: "Multi-Carrier Digital Communications Theory and Applications of OFDM", Second edition, Springer, 2004

[7] Krüger, Mellein: "UMTS Introduction and Measurement", Rohde & Schwarz, 2004

[8] Geßner: "Long Term Evolution – A concise introduction to LTE and its measurements", Rohde & Schwarz, 2011

[9] Rohde & Schwarz: "LTE-Advanced Technology Introduction", Application Note 1MA169, 2010

[10] Foschini, Gans: "On Limits of Wireless Communications in a Fading Environment when Using Multiple Antennas", Wireless Personal Communications 6, 311-335, 1998

About Rohde & Schwarz
Rohde & Schwarz is an independent group of companies specializing in electronics. It is a leading supplier of solutions in the fields of test and measurement, broadcasting, radiomonitoring and radiolocation, as well as secure communications. Established more than 75 years ago, Rohde & Schwarz has a global presence and a dedicated service network in over 70 countries. Company headquarters are in Munich, Germany.

Environmental commitment
- Energy-efficient products
- Continuous improvement in environmental sustainability
- ISO 14001-certified environmental management system

Certified Quality System
ISO 9001

Regional contact
Europe, Africa, Middle East
+49 89 4129 12345
customersupport@rohde-schwarz.com

North America
1-888-TEST-RSA (1-888-837-8772)
customer.support@rsa.rohde-schwarz.com

Latin America
+1-410-910-7988
customersupport.la@rohde-schwarz.com

Asia/Pacific
+65 65 13 04 88
customersupport.asia@rohde-schwarz.com

This application note and the supplied programs may only be used subject to the conditions of use set forth in the download area of the Rohde & Schwarz website.

Rohde & Schwarz GmbH & Co. KG
Mühldorfstraße 15 | D - 81671 München
Phone + 49 89 4129 - 0 | Fax + 49 89 4129 – 13777

www.rohde-schwarz.com